

Spectator field models in light of spectral index after Planck

Takeshi Kobayashi,^{a,b#1} Fuminobu Takahashi,^{c#2} Tomo Takahashi,^{d#3}
and Masahide Yamaguchi^{e#4}

^a *Canadian Institute for Theoretical Astrophysics, University of Toronto,
60 St. George Street, Toronto, Ontario M5S 3H8, Canada*

^b *Perimeter Institute for Theoretical Physics,
31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada*

^c *Department of Physics, Tohoku University, Sendai 980-8578, Japan*

^e *Department of Physics, Saga University, Saga 840-8502, Japan*

^e *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

Abstract

We revisit spectator field models including curvaton and modulated reheating scenarios, specifically focusing on their viability in the new Planck era, based on the derived expression for the spectral index in general spectator field models. Importantly, the recent Planck observations give strong preference to a red-tilted power spectrum, while the spectator field models tend to predict a scale-invariant one. This implies that, during inflation, either (i) the Hubble parameter varies significantly as in chaotic inflation, or (ii) a scalar potential for the spectator field has a relatively large negative curvature. Combined with the tight constraint on the non-Gaussianity, the Planck data provides us with rich implications for various spectator field models.

^{#1}takeshi@cita.utoronto.ca

^{#2}fumi@tuhep.phys.tohoku.ac.jp

^{#3}tomot@cc.saga-u.ac.jp

^{#4}gucci@phys.titech.ac.jp

1 Introduction

The origin of primordial curvature fluctuations is one of the most important issues in cosmology and has been the target of intense study. Although quantum fluctuations of the inflaton are the most plausible candidate from a minimalist point of view, there may be many other light scalar fields in nature, one of which is responsible for the curvature fluctuations as in the curvaton [1] and the modulated reheating [2] and so on. We revisit such spectator field models, specifically focusing on its viability in the new Planck era.

The statistical information of the curvature perturbations can be extracted by evaluating correlation functions. The power spectrum of the curvature perturbations is usually characterized by its amplitude and spectral index, \mathcal{P}_ζ and n_s , which were recently determined by the Planck satellite with unprecedented accuracy as [3],

$$\log(10^{10}\mathcal{P}_\zeta) = 3.089_{-0.027}^{+0.024} \quad (68\% \text{ CL}), \quad (1)$$

$$n_s = 0.9603 \pm 0.0073 \quad (68\% \text{ CL}), \quad (2)$$

from Planck + WMAP polarization data, assuming the standard flat six parameter Λ CDM model. Importantly, the Planck data strongly favors a red-tilted power spectrum, i.e., $n_s < 1$, over the scale-invariant one at more than 5σ level. On the other hand, the bispectrum of the curvature perturbations is characterized by non-linearity parameters for various configurations of the associated three wave numbers. Among them, it is the non-linearity parameter of local type, $f_{\text{NL}}^{\text{local}}$, that is relevant for our purpose. The Planck data put an extremely stringent constraint on $f_{\text{NL}}^{\text{local}}$ as [4]

$$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8 \quad (68\% \text{ CL}). \quad (3)$$

Our primary concern in this paper is to study the implications of those latest Planck results for the spectator field models.

In the spectator field models, there is a light scalar field, σ , in addition to the inflaton which drives inflation, such that its quantum fluctuations account for the total curvature perturbations. The size of the quantum fluctuations is determined by the Hubble parameter during inflation, H_{inf} . If the mass of σ is much lighter than H_{inf} , it hardly evolves during inflation, and therefore, the predicted spectral index is naively expected to be close to unity, since the only source of the scale-dependence is H_{inf} , which however does not evolve significantly during inflation. Specifically, the spectral index in the curvaton scenario is given by [5]

$$n_s - 1 = 2 \frac{\dot{H}_*}{H_*^2} + \frac{2}{3} \frac{V''(\sigma_*)}{H_*^2}. \quad (4)$$

where σ is a curvaton, $V(\sigma)$ the curvaton potential, and H the Hubble parameter. The prime represents the derivative with respect to σ and the asterisk indicates that the variables are evaluated at the horizon exit of the cosmological scales. One can see from the above formula that the spectral index would be indeed close to unity, if the curvature of the potential was much smaller than H_* and if the Hubble parameter during inflation remained almost constant. The same formula also applies to the modulated reheating scenario simply by identifying σ with a modulus that modulates the inflaton decay rate [6]. Compared to the observed red-tilted spectral index (2), however, it is now

clear that such naive estimate is insufficient to describe the Planck data, and we need to consider seriously the implications of the Planck results for the spectator field models.^{#5}

To this end, we first present a simple derivation showing that the above formula (4) is rather robust and holds in a general spectator field model, as long as the light scalar field is responsible for the total curvature perturbations and it does not affect the dynamics of inflation at least until the CMB scales exited the horizon. The spectral index for some other cases are discussed as well. This enables us to derive implications from the Planck result on n_s for various spectator field models such as those studied in Ref. [10]. We also refer the reader to [11] which derived the expression (4) under general situations.

The predicted expression for n_s in a general spectator field model implies that there are basically two choices to realize the observed red-tilted spectral index. The first one is to consider a relatively large variation of the Hubble parameter during inflation. For example, a chaotic inflation based on a quartic potential is able to account for $n_s \simeq 0.96$. The second choice is to assume a relatively large and negative mass squared of the potential. Specifically, a potential with a negative curvature satisfying $V''(\sigma_*) \simeq -0.06H_*^2$ leads to $n_s \simeq 0.96$. As we shall discuss in detail in Sec. 3 and Sec. 4, this places non-trivial constraints on the models. In the curvaton scenario, for instance, the curvaton should come close to dominating the Universe when it decays, in order to satisfy the stringent constraint on non-Gaussianity (3). However this turns out to be quite non-trivial, because of the large negative curvature; the curvaton tends to start to oscillate relatively soon after inflation ends, and it is considered to decay faster for a heavier curvaton mass. Then the curvaton domination may be realized only for high reheating temperature and high-scale inflation. This problem can be circumvented by invoking a hilltop curvaton model [8, 9, 12], where the onset of the oscillation is delayed. The curvaton domination is then allowed with many orders of magnitude of the inflation and reheating scales. Interestingly, however, $f_{\text{NL}}^{\text{local}}$ is generically predicted to be of $\mathcal{O}(10)$ in the hilltop limit [8], which will be in tension with (3). The tension can be ameliorated if the curvaton drives a short duration of inflation before it starts to oscillate [9]. Such argument already shows the rich implications of the Planck results. We will discuss these issues in depth in Sec. 3 and Sec. 4.

Let us mention briefly the implications of (3). It has been often claimed that large non-Gaussianities of the curvature perturbations can be produced in spectator field models such as the curvaton [13–16] and the modulated reheating models [17, 18]. However, in fact, the spectator field models predict a $f_{\text{NL}}^{\text{local}}$ of order unity for a wide (therefore natural) parameter space [19], and so, the constraint on the non-Gaussianity alone does not exclude the spectator field models. As we have seen above, it is very important to study the implications of both (2) and (3) for the spectator field models. The prediction for $f_{\text{NL}}^{\text{local}}$ of order unity should be contrasted to that of the standard single field inflation models in which an inflaton itself is responsible for the curvature perturbations, because the latter predicts $f_{\text{NL}}^{\text{local}}$ suppressed by slow-roll parameters [20, 21]. There is still room for non-zero primordial non-Gaussianity satisfying (3).

Our paper is organized as follows: In the next section, we derive a formula of the spectral index that holds in a general spectator field model. In Sec. 3, we discuss how to accommodate the spectator field models to the Planck results, particularly paying attention to the spectral index of the curvature

^{#5} Note that the red-tilted spectrum index was already favored by the pre-Planck observations such as WMAP [7]. The implications for the curvaton model were studied in Refs. [8, 9].

perturbations. In Sec. 4, we discuss the implications for the curvaton scenario in detail. The last section is devoted to discussion and conclusions.

2 Spectral index in spectator field models

In this section we derive a general expression for the spectral index of the curvature perturbation spectrum produced by spectator fields. Here we define spectator field models as mechanisms where the curvature perturbations are generated by a light scalar σ having no influence on the inflationary expansion while the CMB scales exit the horizon. (Hence we do not consider fluctuations of the inflaton field.) Examples of such models include the curvaton [1], modulated reheating [2], and inhomogeneous end of inflation [22–27].

Using the $\delta\mathcal{N}$ -formalism [28–31], the curvature perturbation ζ can be computed as the difference among different patches of the Universe in the e-folding numbers between an initial flat time-slice when the separate universe assumption is a good approximation, and a final uniform-density time-slice when the Universe is adiabatic:

$$\zeta_{\mathbf{k}} = \frac{\partial\mathcal{N}}{\partial\sigma}\delta\sigma_{\mathbf{k}}, \quad (5)$$

where we have expanded ζ in Fourier space in terms of the field fluctuations to linear order, and the background is assumed to be homogeneous and isotropic. Here we consider the Universe to be an attractor-like system, so that the e-folding number is given as a function of the field value σ at an arbitrary time when σ follows an attractor solution, i.e., $\mathcal{N} = \mathcal{N}(\sigma)$.

Now, let us choose two instants of time: t_* when a certain wave number k (say, the pivot scale) exits the horizon (i.e. $k = aH$), and t_0 when the smallest among the wavelengths of interest (say, the entire CMB scales) exits the horizon. Then the curvature perturbation can be written as

$$\zeta_{\mathbf{k}} = \frac{\partial\mathcal{N}_*}{\partial\sigma_0} \frac{\partial\sigma_0}{\partial\sigma_*} \delta\sigma_{\mathbf{k}*}, \quad (6)$$

where \mathcal{N}_* is the e-folding number between time t_* and the final time-slice at t_f ,

$$\mathcal{N}_* = \int_{t_*}^{t_f} H dt, \quad (7)$$

and for the other quantities the subscripts $*$ and 0 denote values at t_* and t_0 , respectively. Since we have assumed that the field σ has negligible effects on the inflationary expansion while the wave modes of interest exit the horizon, (6) can be rewritten using

$$\mathcal{N}_0 = \int_{t_0}^{t_f} H dt \quad (8)$$

as

$$\zeta_{\mathbf{k}} = \frac{\partial\mathcal{N}_0}{\partial\sigma_0} \frac{\partial\sigma_0}{\partial\sigma_*} \delta\sigma_{\mathbf{k}*}. \quad (9)$$

Note that this expression depends on the wave number k only through $\partial\sigma_0/\partial\sigma_*$ and $\delta\sigma_{\mathbf{k}*}$.

Defining the power spectrum $\mathcal{P}_\zeta(k)$ of the curvature perturbations as

$$\langle\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}(k), \quad (10)$$

then it can be expressed in terms of the power spectrum of the field fluctuations $\mathcal{P}_{\delta\sigma}(k)$ (defined similarly as above) as

$$\mathcal{P}_\zeta(k) = \left(\frac{\partial \mathcal{N}_0}{\partial \sigma_0} \frac{\partial \sigma_0}{\partial \sigma_*} \right)^2 \mathcal{P}_{\delta\sigma_*}(k). \quad (11)$$

We suppose that during inflation the Hubble parameter is nearly constant, and that the light field σ follows the slow-roll attractor under a potential $V(\sigma)$ which is a function merely of σ ,

$$3H\dot{\sigma} \simeq -V'(\sigma), \quad (12)$$

where an overdot represents a time-derivative, and a prime a σ -derivative. Here we are assuming $V' \neq 0$. Then by integrating (12), one obtains

$$\int_{\sigma_*}^{\sigma_0} \frac{d\sigma}{V'(\sigma)} = - \int_{t_*}^{t_0} \frac{dt}{3H}. \quad (13)$$

Here we stress again that the right hand side is independent of σ , hence differentiating both sides in terms of σ_* gives

$$\frac{\partial \sigma_0}{\partial \sigma_*} = \frac{V'(\sigma_0)}{V'(\sigma_*)}. \quad (14)$$

Further using

$$\mathcal{P}_{\delta\sigma_*}(k) = \left(\frac{H_*}{2\pi} \right)^2, \quad (15)$$

then (11) becomes

$$\mathcal{P}_\zeta(k) = \left(\frac{\partial \mathcal{N}_0}{\partial \sigma_0} \frac{V'(\sigma_0)}{V'(\sigma_*)} \right)^2 \left(\frac{H_*}{2\pi} \right)^2, \quad (16)$$

where now its scale-dependence shows up through $V'(\sigma_*)$ and H_* . Thus, using $d \ln k \simeq H_* dt$ and (12), one arrives at a general expression for the spectral index in spectator field models:

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k} = 2 \frac{\dot{H}_*}{H_*^2} + \frac{2}{3} \frac{V''(\sigma_*)}{H_*^2}. \quad (17)$$

This clearly shows that a red-tilted (i.e. $n_s < 1$) spectrum is explained either by a tachyonic potential $V'' < 0$, or a time-varying Hubble parameter $\dot{H} < 0$ during inflation. The expression (17) has also been derived in [11] through a generic treatment of adiabatic/isocurvature fluctuations.

The above result can also be extended to cases with multiple light fields σ^a ($a = 1, 2, \dots$), where each field has no effect on the inflationary expansion while the scales of interest exit the horizon. Further assuming that the potential of each field is decoupled from the others, i.e. $V = \sum_a V_a(\sigma^a)$, and also that the fluctuations between different fields have no correlations, one can check that

$$\mathcal{P}_\zeta(k) = \sum_a \left(\frac{\partial \mathcal{N}_0}{\partial \sigma_0^a} \frac{V'_a(\sigma_0^a)}{V'_a(\sigma_*^a)} \right)^2 \left(\frac{H_*}{2\pi} \right)^2, \quad (18)$$

where now a prime on $V_a(\sigma^a)$ denotes differentiation in terms of σ^a . Hence introducing

$$q^a \equiv \left(\frac{\partial \mathcal{N}_0}{\partial \sigma_0^a} \frac{V'_a(\sigma_0^a)}{V'_a(\sigma_*^a)} \right)^2 \bigg/ \sum_b \left(\frac{\partial \mathcal{N}_0}{\partial \sigma_0^b} \frac{V'_b(\sigma_0^b)}{V'_b(\sigma_*^b)} \right)^2, \quad (19)$$

(which satisfies $\sum_a q^a = 1$), one obtains the expression

$$n_s - 1 = 2 \frac{\dot{H}_*}{H_*^2} + \frac{2}{3} \sum_a q^a \frac{V_a''(\sigma_*^a)}{H_*^2} \quad (20)$$

for multiple spectator fields.

Similar analyses can be carried out also for potentials $V(\sigma, t)$ that explicitly depend on time, especially when it takes a separable form [32] during inflation,^{#6} i.e.,

$$V(\sigma, t) = v(\sigma)f(t). \quad (21)$$

(A good example is a Hubble-induced mass term $V \propto H^2 \sigma^2$.) Then given that the time-dependent part varies slowly

$$\left| \frac{\dot{f}}{Hf} \right| \ll 1, \quad (22)$$

the light field (i.e. $|v''f/H^2| \ll 1$) follows the slow-roll approximation (12) and now (13) can be rewritten as

$$\int_{\sigma_*}^{\sigma_0} \frac{d\sigma}{v'(\sigma)} = - \int_{t_*}^{t_0} \frac{f dt}{3H}. \quad (23)$$

If $f(t)$ is unaffected by σ (for e.g. when $f(t)$ arises from a Hubble-dependence), then so is the right hand side of (23), and thus one reproduces the same expression as (17):

$$n_s - 1 = 2 \frac{\dot{H}_*}{H_*^2} + \frac{2}{3} \frac{v''(\sigma_*)f(t_*)}{H_*^2}. \quad (24)$$

3 Red-tilted spectrum in spectator field models

In this section, we discuss how we can construct spectator field models such as the curvaton and the modulated reheating scenarios consistent with the Planck data, in particular, from the view point of the spectral index n_s . As mentioned in the introduction, models with a light spectator field tend to give a rather scale-invariant spectral index, which is now significantly away from the Planck value. In this section, we discuss two possibilities to directly explain the spectral index $n_s = 0.9603 \pm 0.0073$ by the Planck results in the context of the spectator field models: (A) Relatively large variation of the Hubble parameter (B) Negative mass squared of the potential.

3.1 Variation of the Hubble parameter

If the first term of the right hand side in Eq. (17), that is, if $2\dot{H}_*/H_*^2$ is non-negligible, the spectral index can deviate from unity. When inflation is driven by a canonical slow-rolling inflaton, one can rewrite as

$$2 \frac{\dot{H}_*}{H_*^2} = -2\epsilon = -\frac{1}{M_{\text{pl}}^2} \left(\frac{d\phi}{dN} \right)^2, \quad (25)$$

^{#6}The potential can further have time-independent terms that are negligible during inflation, but play important roles in the post-inflationary epoch.

where ϕ is the inflaton, N is the e-folding number, ϵ is the slow-roll parameter, and $M_{\text{pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Thus, if this term is responsible for $1 - n_s \simeq 0.04$, $\Delta\phi \simeq \sqrt{1 - n_s} N M_{\text{pl}} > M_{\text{pl}}$ for $N \sim 60$, which requires large field inflation. Actually, in case of chaotic inflation with the potential $V(\phi) \propto \phi^p$, $2\epsilon \simeq p^2 M_{\text{pl}}^2 / \phi^2 \simeq p / (2N)$. Thus, $n_s \simeq 0.96$ implies $p \sim 2(1 - n_s)N \sim 4$ for $N \sim 60$. Thus, the chaotic inflation with a quartic potential can nicely explain the observed red-tilted spectral index.^{#7} This should be contrasted to the fact that the chaotic inflation with a quartic potential is already strongly disfavored in the case where the inflaton is responsible for the density perturbations because of too large tensor-to-scalar ratio r . In spectator field models, however, the tensor-to-scalar ratio is suppressed (or equivalently, the curvature perturbations are enhanced), making such inflation models consistent with the observations.^{#8}

Finally, let us comment on k-inflation with $\mathcal{L} = K(\phi, X)$ [36]. The far right hand side in Eq. (25) is multiplied by $K_{,X}$, that is, $-2\epsilon = -(K_{,X}/M_{\text{pl}}^2)(d\phi/dN)^2$. Here $X = (d\phi/dt)^2/2$ and $K_{,X}$ is the derivative with respect to X . Therefore, if $|K_{,X}| \gg 1$, large field inflation may not necessarily be required. However, for example, in case of the DBI inflation [37], $K_{,X} = c_s^{-1}$ and the sound velocity c_s is now strongly constrained from the non-Gaussianity as $c_s^{\text{DBI}} \geq 0.07$ (95% CL) [4]. Therefore, large variation comparable to the Planck scale is still required, which is difficult to realize from the microscopic view point [38–40].

3.2 Negative mass squared

Let us next consider the case where the second term in the right hand side of Eq. (17), i.e. $2V''(\sigma_*)/(3H_*^2)$, is responsible for the deviation of the spectral index from the scale invariant one. In this case, the observed red-tilted spectrum implies a negative mass squared of the effective scalar potential, $V''(\sigma_*) < 0$. More concretely, $n_s \simeq 0.96$ can be realized if $V''(\sigma_*) \simeq -0.06H_*^2$.

Here, we discuss implications of the negative mass squared for the spectator field models. For the moment we assume that the scalar potential $V(\sigma)$ does not depend on time. The known spectator field models can be broadly classified into two cases, depending on whether the spectator field fluctuations are converted into curvature perturbations before or after the commencement of oscillations of σ . The former includes the modulated reheating [2], the inhomogeneous end of inflation [22–27], the modulated trapping [41], velocity modulation [42], and so on (i.e. almost all the spectator field models except the curvaton model), while the latter contains the curvaton mechanism as well as some realization of the modulated reheating.

In the former class of models, the spectator field should not start to oscillate until a certain point, as the resultant curvature perturbations would be strongly suppressed, otherwise. Suppose that it starts to oscillate when the Hubble parameter becomes comparable to the absolute magnitude of the effective mass, $H^2 \sim |V''|$. Then, the required relation $V''(\sigma_*) \simeq -0.06H_*^2$ implies that the spectator field σ starts to oscillate relatively soon after inflation. This does not place any stringent constraints on the scenario of inhomogeneous end of inflation. On the other hand, however, the modulated reheating scenario is severely constrained, because the reheating should be completed soon after inflation, before the modulus starts to oscillate. This implies that the reheating process

^{#7} It is interesting to notice that such chaotic inflation based on a quartic potential naturally appears by the use of D-term in supergravity [33], though it is realized by F-term as well [34].

^{#8} Constraints on inflation models in spectator field models have been investigated in [35].

must be extremely efficient, which results in rather high reheating temperature for most of inflation models. Further, it is unclear whether it is possible to induce such an efficient reheating process via a usual perturbative decay of the inflaton, on which most of the calculations of the modulated reheating are based.

It should be noticed that the above discussion is based on the assumptions that the scalar potential of the spectator field is time-independent and it starts to oscillate when $H^2 \sim |V''|$. If these assumptions are relaxed, the commencement of oscillations can be delayed, avoiding the above constraint. First, consider a time-dependent scalar potential. For example, the spectator field may receive a negative Hubble mass correction, $-(k/2)H^2\sigma^2$, from interactions with the inflaton, where $k > 0$ is a numerical coefficient. Indeed, such corrections arise ubiquitously in supergravity. If k is about 0.06, the observed spectral index can be explained. Importantly, the Hubble mass decreases after inflation. After reheating, the inflaton contribution to the Hubble mass disappears, but instead, a Hubble induced mass with $k = \mathcal{O}(10^{-2})$ generically arises from Planck-suppressed couplings with the standard-model particles that constitute the thermal plasma [43]. Therefore, the early oscillations of the spectator field can be avoided, if the zero-temperature potential has a curvature much smaller than the Hubble parameter during inflation. Next let us consider a hilltop initial condition for the spectator field. As long as the spectator field sits sufficiently near the top of the potential ^{#9}, the onset of oscillations will be delayed. This can be understood as follows. For the hilltop initial condition, the spectator field starts to oscillate when

$$\left| \frac{\dot{\sigma}_{\text{osc}}}{H_{\text{osc}}(\sigma_{\text{osc}} - \sigma_{\text{min}})} \right| \sim 1, \quad (26)$$

where the potential minimum is located at $\sigma = \sigma_{\text{min}}$, and we assume that the potential monotonically increases from the potential minimum to the maximum. Under the slow-roll approximation, the above relation can be rewritten as

$$H_{\text{osc}}^2 \sim \left| \frac{V'(\sigma_{\text{osc}})}{(\sigma_{\text{osc}} - \sigma_{\text{min}})} \right|. \quad (27)$$

Therefore, the onset of the oscillation of a spectator field σ is significantly delayed compared to the naive expectation, if

$$|V'(\sigma_{\text{osc}})| \ll |(\sigma_{\text{osc}} - \sigma_{\text{min}})V''(\sigma_{\text{osc}})|. \quad (28)$$

This condition is satisfied if σ sits near the top of the potential. Thus, the hilltop initial condition relaxes the constraint on the spectator models. ^{#10}

The representative example of the latter class of the spectator field models is the curvaton model [1]. The curvature perturbations are generated effectively when the curvaton starts to (or at least comes close to) dominate the Universe, and then the curvaton decay produces a significant amount of entropy and radiation. Importantly, it is the density perturbations, $\delta\rho_\sigma/\rho_\sigma$, that is relevant for evaluating the resultant curvature perturbations. Note that both $\delta\rho_\sigma$ and ρ_σ decrease after the onset of oscillations. This is the reason why the curvaton model works well even though the conversion takes place after the onset of the curvaton oscillation. In order to estimate the final

^{#9} If it sits too close to the top of the potential, it may lead to the formation of topological defects.

^{#10} In principle it is possible to combine the two possibilities mentioned (i.e., negative Hubble-induced mass + hilltop initial condition).

curvature perturbations, however, it is necessary to follow the evolution of the curvaton from during inflation until its decay, which requires a dedicated analysis in case of the negative mass squared. Therefore, we devote the next section to the detailed analysis of this case.

Finally let us briefly mention another example in the latter class of the spectator field models. Usually it is assumed that the modulus does not start to oscillate until reheating in the modulated reheating scenario. However, some realization of the model may work even if the modulus starts to oscillate before the reheating. This is the case if the inflaton decay rate is proportional to some power of σ . For instance, suppose that the inflaton decays through an interaction whose coefficient is proportional to σ , in which case the inflaton decay rate is proportional to σ^2 . As long as the decay mode gives dominant contribution to the total decay rate, and given that the fluctuation of the decay rate is determined by the oscillation amplitude, the resultant curvature perturbation $\zeta \sim \delta\Gamma/\Gamma \propto \delta \ln \sigma_{\text{amp}}^2$ is not significantly suppressed even after the commencement of oscillations. In this case, we need to follow the evolution of the modulus during from inflation until reheating in order to estimate the final curvature perturbation. To this end, the technique used in the curvaton scenario, which we shall discuss in the next section, will be useful, and in fact it can be applied to this class of the modulated reheating in a straightforward way. Note that the problems associated with the early oscillations can be avoided in this case, allowing lower reheating temperature.

4 Planck constraints on curvaton scenarios

In the curvaton scenario [1], the spectator field σ (which we refer to as the curvaton in this section) undergoes oscillations around its potential minimum in the post-inflationary era. After the inflaton decays, the curvaton's energy density increases relative to the background radiation, and thus the curvaton generates curvature perturbations as it comes close to dominating the Universe. The curvaton is assumed to eventually decay into radiation.

As is seen in (17), the curvaton needs to lie along a negatively curved potential during inflation with an effective mass not much smaller than the inflationary Hubble parameter,

$$V''(\sigma_*) \sim -10^{-2} H_*^2, \quad (29)$$

in order to produce the red-tilt (2), if not from large-field inflation. However, unlike other spectator mechanisms the curvaton generates perturbations as it oscillates, thus issues discussed in the previous section are not necessarily problems here. In this section we will see instead that the Planck constraints on non-Gaussianity impose rather strict requirements for curvaton model building.

4.1 Quadratic curvatons

Let us start by discussing the simplest case, namely curvatons with a quadratic potential,

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2. \quad (30)$$

Although a quadratic curvaton itself cannot produce a red-tilt, this simple model captures some basic properties that are shared by a rather wide group of curvaton potentials. (For e.g., curvatons with cosine-type potentials (36) behave similarly to quadratic ones except for when σ_* is very close

to the potential maximum.) Hence the discussions in this subsection also apply to curvatons whose overall potential forms do not drastically deviate from a quadratic one.

Denoting the energy density ratio between the curvaton and radiation (originating from the inflaton) upon curvaton decay by

$$\hat{r} \equiv \left. \frac{\rho_\sigma}{\rho_r} \right|_{\sigma\text{-decay}}, \quad (31)$$

the power spectrum and non-Gaussianity parameter from a quadratic curvaton takes the form

$$\mathcal{P}_\zeta = \left(\frac{2\hat{r}}{4+3\hat{r}} \frac{H_*}{2\pi\sigma_*} \right)^2, \quad (32)$$

$$f_{\text{NL}} = \frac{5}{12} \left(-3 + \frac{4}{\hat{r}} + \frac{8}{4+3\hat{r}} \right). \quad (33)$$

The Planck constraint on f_{NL} (3) requires the curvaton to (almost) dominate the Universe before decaying, i.e.,

$$\hat{r} \gtrsim 0.1. \quad (34)$$

A quadratic curvaton starts to oscillate when the Hubble parameter becomes comparable to its mass, i.e. $H_{\text{osc}} \sim m_\sigma$. (Hereafter we use the subscript “osc” to denote quantities at the onset of the curvaton oscillation). Then, ignoring the time evolution of the curvaton field and using $\sigma_{\text{osc}} \sim \sigma_*$, the curvaton energy fraction at $t = t_{\text{osc}}$ is

$$\left. \frac{\rho_\sigma}{\rho_{\text{total}}} \right|_{\text{osc}} \simeq \frac{V(\sigma_{\text{osc}})}{3M_p^2 H_{\text{osc}}^2} \sim (10^3\text{--}10^6) \left(\frac{H_*}{M_p} \right)^2 \lesssim 10^{-3}, \quad (35)$$

where we have also used (32), (34), and $\mathcal{P}_\zeta \approx 2.2 \times 10^{-9}$. The far right hand side is due to the upper bound on the inflationary scale from the constraint on the tensor-to-scalar ratio $r < 0.11$ (95% CL) [3]. One sees from (35) that the energy fraction at the onset of the oscillations is bounded by H_* , in other words, inflation with lower scales requires the curvaton to oscillate for larger numbers of e-foldings before dominating the Universe. Therefore the curvaton scenario can be successful provided that the inflation and reheating (= inflaton decay) scales are high and/or the curvaton decay rate is low.

Here it should be noted that the curvaton decay rate typically scales as some positive power of the oscillation mass. Unless the oscillation mass is designed to be significantly smaller than the curvaton’s effective mass during inflation (29), the observed value of the spectral index (2) requires a rather large curvaton mass and thus prevents the curvaton decay rate from being tiny. Whether this issue becomes a serious problem depends on the details of the model, however this was demonstrated in [8] to impose severe constraints when the curvaton is a pseudo-Nambu-Goldstone boson of a broken U(1) symmetry, possessing a cosine-type potential

$$V(\sigma) = \Lambda^4 \left[1 - \cos \left(\frac{\sigma}{f} \right) \right]. \quad (36)$$

Interactions of such an axionic curvaton with other particles are typically suppressed by the symmetry breaking scale f , and thus in terms of the effective mass at the minimum $m_\sigma = \Lambda^2/f$, its decay rate reads

$$\Gamma_\sigma \sim \frac{m_\sigma^3}{f^2} = \frac{\Lambda^6}{f^5}. \quad (37)$$

For a large effective mass of (29), the axionic curvaton can dominate the Universe and produce the primordial perturbations only if the inflationary scale and reheating (= inflaton decay) temperature are as high as $H_{\text{inf}}, T_{\text{reh}} \gtrsim 10^{13}\text{GeV}$ (almost saturating the current bound on primordial tensor modes), or if the initial curvaton field value is located very close to the hilltop $\sigma_* \approx \pi f$ such that the system is no longer approximated by a quadratic potential. This latter case is discussed later.

In summary, high inflation and reheating scales and/or suppression of the curvaton decay rate are required in order for curvatons following the familiar relations

$$\mathcal{P}_\zeta \sim \left(\frac{H_*}{\sigma_*}\right)^2, \quad H_{\text{osc}} \sim m_\sigma, \quad (38)$$

to dominate the Universe and produce perturbations consistent with observations.

4.2 Possible curvaton scenarios with negative mass squared

Facing the situation noted above, we lay out possible ways to reconcile the curvaton mechanism with observations without relying on the inflaton sector. The basic ideas are to extend the curvaton lifetime and/or enhance the curvaton energy fraction prior to the oscillations.

- *Suppressing the decay rate.* The curvaton can dominate the Universe and produce nearly Gaussian perturbations if its decay rate is highly suppressed, for e.g., if the decay rate vanishes at tree level, or is helicity suppressed. Furthermore, a curvaton potential whose effective mass around its minimum is much smaller than that at σ_* can suppress the decay rate.
- *Potential with a flat plateau.* If the negatively curved region of the potential around σ_* during inflation and the potential minimum is connected by a flat plateau, then the curvaton would undergo a period of slow-roll and delay the onset of the oscillations. This can enhance the curvaton energy fraction.
- *Negative Hubble-induced mass squared.* Curvatons that obtain negative Hubble-induced mass terms during inflation can produce a red-tilted perturbation spectrum. Furthermore, the Hubble-induced mass decreases/vanishes in the post-inflationary era and thus the onset of the curvaton oscillations is delayed.
- *Hilltop curvaton.* The onset of oscillations is delayed for a curvaton that lies close to a local maximum of its potential (cf. Sec. 3.2). Furthermore, hilltop potentials significantly enhance the linear perturbations [8], and thus violate both relations in (38). However, the latest f_{NL} bound from Planck imposes rather strict constraints on such hilltop curvaton scenarios. Let us elaborate on this case in the next subsection.

4.3 Hilltop Curvatons

A simple way to resolve the issues discussed in Sec. 4.1 and give a tachyonic curvaton mass (29) is to have the curvaton located close to a local maximum of its potential during inflation, i.e., the potential well approximated by

$$V(\sigma) = V_0 - \frac{1}{2}m_\sigma^2(\sigma - \sigma_1)^2 \quad (39)$$

until the curvaton starts to oscillate (for e.g. the $\sigma_* \rightarrow \pi f$ limit of the axionic curvaton). The hilltop potential produces a red-tilted perturbation spectrum, and since the potential flattens out in the hilltop limit the onset of the curvaton oscillation is delayed to $H_{\text{osc}}^2 \ll m_\sigma^2$. Furthermore it is known that such flattened potentials lead to an inhomogeneous onset of the curvaton oscillation, and as a consequence strongly enhance the resulting curvature perturbations [8, 9]. Thus hilltop curvatons simultaneously violate the relations in (38) and can dominate the Universe without requiring high scale inflation or suppression of Γ_σ .^{#11}

Let us set the potential minimum (existing outside the field range where (39) is a good approximation) to $\sigma = 0$, and without loss of generality assume $0 < \sigma_{\text{osc}} < \sigma_* < \sigma_1$. Then in the hilltop region:

$$\sigma_{\text{osc}} \gg \sigma_1 - \sigma_{\text{osc}}, \quad V_0 \gg m_\sigma^2 (\sigma_1 - \sigma_{\text{osc}})^2, \quad (40)$$

(the first condition corresponds to (28)) the power spectrum of linear perturbations becomes

$$\mathcal{P}_\zeta \simeq \left(\frac{3\hat{r}}{4 + 3\hat{r}} \frac{\sigma_1 - \sigma_{\text{osc}}}{\sigma_1 - \sigma_*} \frac{H_*}{2\pi\sigma_{\text{osc}}} \right)^2, \quad (41)$$

where \hat{r} is defined in (31). The field values during inflation and at the onset of the oscillations are related by

$$\log \left(\frac{\sigma_1 - \sigma_*}{\sigma_1 - \sigma_{\text{osc}}} \right) \simeq -A \frac{\sigma_{\text{osc}}}{\sigma_1 - \sigma_{\text{osc}}}, \quad (42)$$

where A is constant of order unity depending on whether reheating happens before/after the curvaton starts to oscillate. The left hand side of (42) being a log term shows that σ_{osc} is insensitive to σ_* , i.e., as one takes the hilltop limit $\sigma_* \rightarrow \sigma_1$, the value of σ_{osc} approaches σ_1 much slower than σ_* does. Hence $(\sigma_1 - \sigma_{\text{osc}})/(\sigma_1 - \sigma_*) \gg 1$ and the power spectrum (41) is enhanced compared to the quadratic case (32).

However, we should also remark that the non-Gaussianity also mildly increases in the hilltop limit as

$$f_{\text{NL}} \simeq \frac{5(4 + 3\hat{r})}{18\hat{r}} \frac{\sigma_{\text{osc}}}{\sigma_1 - \sigma_{\text{osc}}}, \quad (43)$$

which is clearly much greater than unity from (40) even when $\hat{r} \gg 1$. (For e.g., the typical value for f_{NL} from an axionic curvaton at the hilltop is a few tens [8, 9].)

Thus we have seen that hilltop models that are free from the issues discussed in Sec. 4.1 are now strictly constrained from the latest f_{NL} bounds provided by Planck. We also note that the increase of non-Gaussianity towards the hilltop is a rather generic feature of hilltop spectator field models where the curvature perturbations are generated after the field starts oscillating. (However, if the perturbations are produced before the oscillations (as in most modulated reheating scenarios), then spectator fields at the hilltop do not necessarily lead to large f_{NL} .)

Before concluding this section, let us remark that the f_{NL} constraints on hilltop curvatons can be resolved if the curvaton initially lies extremely close to the potential maximum such that it dominates the Universe in the post-inflationary era before the oscillations. Such curvatons driving a (short) second inflationary epoch tend to generate small f_{NL} (say, of order unity) compatible with observations. Detailed discussions on inflating curvatons can be found in, e.g. appendix of [9].

^{#11}On the other hand, steep potentials such as in self-interacting curvatons [44–47] tend to suppress the resulting perturbations compared to quadratic cases. This further suppresses the energy fraction upon decay (35).

5 Discussion and conclusions

We have discussed implications of recent Planck results for spectator field models, such as the curvaton, modulated reheating and so on, which have been often referred as models generating large (local-type) non-Gaussianity. It should be noted that spectator field models of these kinds can also predict $f_{\text{NL}} \sim \mathcal{O}(1)$ in wide range of their parameter space, thus, although Planck data now very severely constrains a non-linearity parameter f_{NL} , they are not excluded only by the argument of non-Gaussianity. Importantly, the spectral index n_s is also precisely measured by Planck and a red-tilted one is strongly favored. Since spectator field models tend to give a scale-invariant power spectrum, the consideration of the spectral index, in combination with f_{NL} , gives significant implications for spectator field models.

Based on the formula (4) presented in Sec. 2, we discussed two possible scenarios to have a red-tilted spectral index consistent with Planck results: (i) large variation of Hubble parameter during inflation as in the large field inflation model (ii) a spectator field potential with a relatively large negative mass squared. The former one resorts to the inflation model, on the other hand, the latter one relies on the spectator field sector to realize a red-tilted spectrum. Some possible scenarios for the latter case were discussed in Sec. 3.2. Furthermore, we have given detailed discussions for the curvaton model and argued that a hilltop curvaton can be viable, giving a red-tilted spectral index and $f_{\text{NL}} \sim \mathcal{O}(1)$, when the curvaton field initially lies extremely close to the potential maximum to give a second inflationary epoch. Other ways to reconcile the curvaton mechanism with observations were proposed in Sec. 4.2.

Lastly we briefly comment on yet another possibility having a successful spectator field model. In fact, the constraint on the spectral index given in (2) can be changed when we consider an extension of the concordance cosmological model, the standard Λ CDM model. The effective number of neutrino species is usually set to be $N_{\text{eff}} = 3.046$. However, if we add extra relativistic degrees of freedom to vary N_{eff} , a scale-invariant power spectrum may give a equally good fit to the observations. According to the Planck analysis on the one-parameter extensions, the scale-invariant power spectrum is marginally allowed at 2σ level [48]. From theoretical view point, there are a lot of candidates of dark radiation [6, 49]. In particular, the present authors showed that dark radiation is naturally produced in the context of the modulated reheating scenario [6]. In the scenario, the decay rate of the inflaton must depend on a modulus, which in turn implies that the inflaton must couple to the modulus and hence a significant amount of the modulus is produced through the inflaton decay. We also note that an extension with freely varying Y_p may also allow a scale-invariant spectrum. A large value of the Helium abundance Y_p can give a similar effect as in the case of the extra radiation discussed above. By varying Y_p freely, the scale-invariant spectrum is marginally allowed at 2σ level [48].

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